Force-Free Collisonless Current Sheets:
A Systematic Method for Adding Asymmetries

School of Mathematics and Statistics, University of St Andrews, St Andrews KY16 9SS, United Kingdom
Lilli Nadol (email: lmn6@st-andrews.ac.uk)
Thomas Neukirch (email: tn3@st-andrews.ac.uk)
I. Y. Vasko (University of California; Space Research Institute of Russian Academy of Sciences)
A. V. Artemyev (University of California; Space Research Institute of Russian Academy of Sciences)
O. Allanson (University of Reading)

Solar & Magnetospheric Theory Group
University of St Andrews

University of St Andrews

I. Introduction

- Recent observations have shown that current sheets in the solar wind have systematic asymmetries in their particle density and temperature while the pressure remains constant. (e.g. Artemyev et al. (2019))

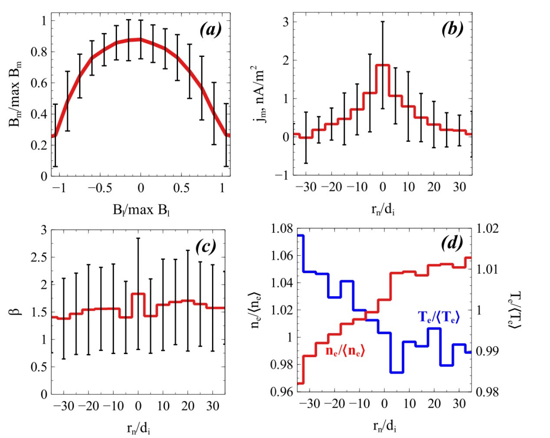
- Equilibrium distribution functions f_{ff} are known for force-free current sheets in the solar wind with resulting density and temperature that are constant or at most varying symmetrically in space.

- Neukirch et al. (2020) have shown that temperature and density asymmetries can be modelled using a distribution function of the form $f = f_{ff} + \Delta f$.

II. Questions

- Does a systematic mathematical method exist to determine such asymmetric equilibria?

- If such a method exists, can it be used to find physically reasonable distribution functions?



Average profiles of magnetic field, current density, and plasma characteristics for a data set of ∼200 discontinuities observed by the ARTEMIS spacecraft in the near-Earth solar wind (see details of the data set in Artemyev et al. (2019)).

The main criterion of discontinuity selection to the data set is the peak current density exceeding $1\ nA\ m^{-2}$. Black error bars show the standard deviation. In each case, electron densities and temperatures are normalized by the average value across the discontinuity. Orientation of r_n is chosen to have $\frac{dn_e}{dr_n} > 0$ for all selected discontinuities.

(Graph from Neukirch et al. (2020))

III. This Poster

- We introduce the underlying mathematical basis and methodology for asymmetric particle distribution functions.
- We present our results regarding the equilibrium distribution functions originally presented by Neukirch et al. (2020).
- We explore different approaches to find new examples and give an outlook on future work.

Basic Theory

- We want to model the current sheet as stationary and assume spatial symmetry in two dimensions (here x and y). Due to this one-dimensional nature of the current sheet models one can find equilibrium distribution functions of the Hamiltonian H and the canonical momenta p_x and p_y:

$$f \equiv f(H, p_x, p_y)$$

- We assume quasi-neutrality.
- We want to construct equilibrium distribution functions of the form $f = f_{ff} + \Delta f$ where following Neukirch et al. (2020) $\Delta f \equiv \Delta f(H, p_x)$ has been added in order to accommodate for the asymmetric contributions to density and temperature.

Asymmetric Contribution

We want Δf to contribute to the number density but not to the current density. This leads to the conditions:

$$n_{\Delta f} = \int \Delta f d^3 v \neq 0 \qquad \int v_x \Delta f d^3 v = 0$$

These can be written as:

$$n_{\Delta f} = \frac{\pi}{m^2} \int_{-\infty}^{\infty} \int_{H_{min}}^{\infty} \Delta f dH dp_x \neq 0$$

$$0 = \int_{-\infty}^{\infty} \int_{H_{min}}^{\infty} \frac{\partial \Delta f}{\partial p_x} (\bar{H}_{min} - H) dH dp_x$$

Separable Approach

$$\Delta f \equiv \Delta f(H, p_x) = g(H) k(p_x)$$

$$\frac{\partial \Delta f}{\partial p_x} = g(H) k'(p_x)$$

such that the condition on the current density is given by

$$0 = \int_{-\infty}^{\infty} k'(p_x) \int_{H_{min}}^{\infty} (H - \bar{H}_{min}) g(H) dH dp_x$$

Fourier Transform Method

Setting $g(H) = G''(H)$ and imposing the boundary conditions $\{G(H), G'(H), HG'(H)\} \to 0$ as $H \to \infty$ the above condition can be reduced to a convolution type integral. Using Fourier transformation this integral can be reduced to the product of two functions. Hence, our condition is given by this product vanishing.

Approaches for Determining g and k

1. Initially choose $g(H)$ (or $k(p_x)$). Then using Fourier transformation we can determine one of the above Fourier transforms. The other function can be chosen such that their product vanishes. Using inverse Fourier transformation, the pair g and k can be determined.
2. Choose two functions as Fourier transforms whose product is zero and whose inverse Fourier transforms exist. Using inverse Fourier transformation one can obtain g and k.

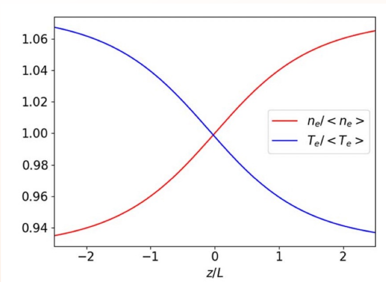
Examples from Neukirch et al. (2020) Verified with Fourier Method

$$k_1(p_x) = C_1 p_x + C_o \qquad g_1(H) = K\left(e^{-aH} - ce^{-bH}\right)$$

$$k_2(p_x) = \frac{1}{\omega}\sin(\omega p_x) \qquad g_2(H) = K(a - bH)e^{-bH}$$

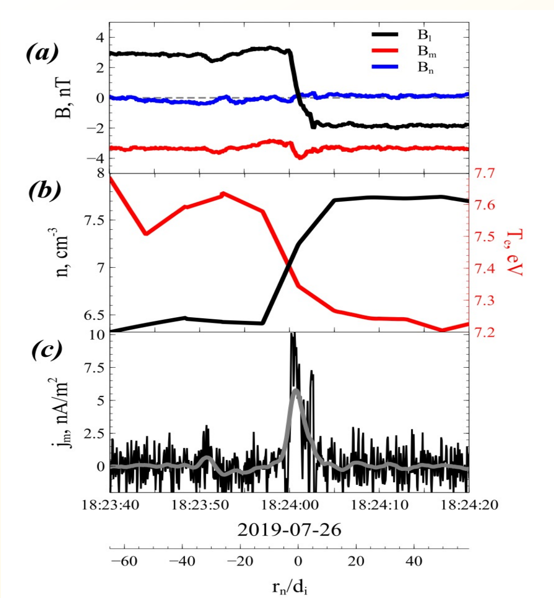
$$k_3(p_x) = \frac{1}{\omega}\exp(\omega p_x)$$

Pairs of theses functions work for specific choices of a, b and c.



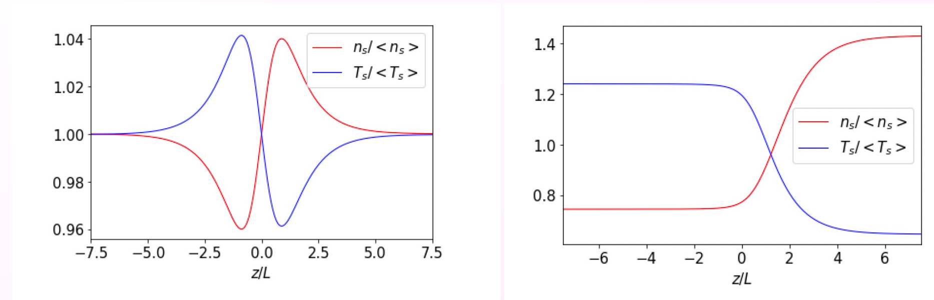
Asymmetric density and temperature profiles resulting from the theoretical models using k_1 and g_1.

(Graph from Neukirch et al. (2020))



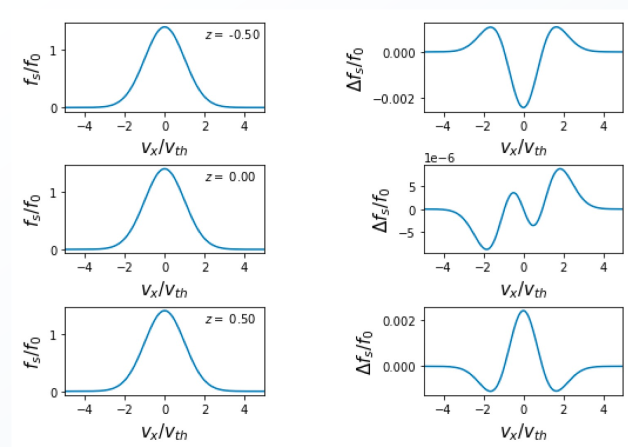
Example of a current sheet crossing by ARTEMIS spacecraft: (a) three magnetic field components in the local coordinate system with the additional constraint $< B_n > = 0$, (b) electron density and temperature measurements, and (c) current density profile (gray color shows smoothed profile). Bottom horizontal axis shows the spatial coordinate across the sheet (normalized on the ion inertial length, d_i).

(Graph from Neukirch et al. (2020))



Asymmetric density and temperature profiles resulting from the theoretical models using k_2 and k_3(both are the same for $g_1$and g_2). We notice that these examples work well mathematically but resulting temperature and density profiles do not match as well with observations as the linear case (e.g. compare to density and temperature measurements on the right).

Illustrative Example



In the left column the dependence of the full particle distribution function $f = f_{ff} + \Delta f$ on v_x (for $v_y = v_z = 0$) is shown at three different positions $\mathrm{z/L} = -0.5$(top row), $\mathrm{z/L} = 0.0$ (middle row) and $\mathrm{z/L} = 0.5$ (bottom row) for k_2 and g_2. The right column shows the same plots for Δf alone.

Finding New Examples: Choosing $g(H)$

When choosing $g(H) = \delta(H_0 - H)$ an infinite number of possible choices for k exist, one of which is given by k_2. Unfortunately, this case does not lead to physically reasonable functions as the distribution function locally attains negative values due to the derivatives of the delta dirac function in the expressions for G, G' and HG'. This problem can be solved by adding a positive constant, but then the background density does not vanish.

Finding New Examples: Choosing the Fourier Transforms

For the first Fourier transform we chose a combination of a step function and and exponential, for the second we chose a combination of a step function and a delta dirac function. This leads to

$$k(p_x) = \frac{1}{2}\left(\theta(p_x) + \frac{1}{i\pi}\mathrm{Ei}(2i\pi u_1 p_x)\right)$$

$$g(H)$$
$$= 2\pi^{7/2}m^2\exp(-2\pi^2 mH)\{\pi - 1 + \mathrm{erf}(\pm\pi i\sqrt{2mH} - u_0)\}$$
$$\mp\frac{m}{H}\exp(\pm 2i\pi u_0\sqrt{2mH} - u_0^2)\{\pi i\sqrt{2mH} + \frac{i}{4\pi i H^2} + u_0\}$$

Where the signs depend on the value of p_x. Again, the necessary boundary conditions are not fulfilled by g and the calculation of the number density and pressure tensor require the integration of products of $\exp(x)$, $\mathrm{erf}(x)$ and $\mathrm{Ei}(x)$ which needs to be done numerically.

Results

- Choosing trigonometric and exponential functions for k seems to produce the most reasonable results for k from a mathematical point of view but resulting density and temperature profiles do not represent observations well.
- We have been able to find examples that are mathematically viable which do not result in distribution functions that are physically reasonable (e.g. see example on the right).

Discussion and Outlook

- Distribution functions $f \equiv f(H, p_x, p_y) = f_{ff} + \Delta f$ with added asymmetric contributions of the form $\Delta f \equiv \Delta f\ (H, p_x)$ can not accommodate for all possible 1D cases. Relaxing the assumptions that we have made will lead to other possible forms of Δf and f.
- We assumed the asymmetric contribution to be independent of the canonical momentum in $y-$ direction. Hence, including p_y leads to new possible choices of $\Delta f \equiv \Delta f\ (H, p_x, p_y)$.
- We have only looked into Δf of separable form. Relaxing this assumption leads to other new possibilities.

References

1. Artemyev, A., Angelopoulos, V. & Vasko, I. (2019), 'Kinetic properties of solar wind discontinuities at 1 AU observed by Artemis', Journal of Geophysical Research: Space Physics, 124(6), 3858–3870. URL: https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/2019JA026597
2. Neukirch, T., Vasko, I., Artemyev, A. & Allanson, O. (2020), 'Kinetic models of tangential discontinuities in the solar wind', The Astrophysical Journal 891(1), 86. URL: http://dx.doi.org/10.3847/1538-4357/ab7234